The structure of gold and silver spread returns

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Abstract
The price dynamics of gold and silver have long been a matter of popular concern and fascination. The objective of this study is to investigate the dynamics of the bivariate relationship between gold and silver prices. First, we investigate the spread, measured as the price difference between gold and silver trading as a futures contract. Then the presence of a fractal structure is measured using statistical techniques based on rescaled range analysis after accommodating short-term autocorrelated innovations in the return process. To highlight the economic consequences of fractality, we apply trading rules based upon the Hurst coefficient to the data. Importantly, we find that these rules outperform simple buy-hold and moving-average strategies over varying holding periods.

Key words: Long term dependence; volatility; gold silver spread; futures; fractal structure; Hurst coefficient
The structure of gold and silver spread returns

1. Introduction
The price dynamics of precious metals, generally, and gold and silver in particular, has long been a matter of popular concern and fascination. Recently, a number of authors have successfully modelled the stochastic nature of precious metal returns (e.g. Urich, 2000; Casassus and Collin-Dufresne, 2005), while new tools from econometrics have demonstrated important insights into the long term price relationships between bivariate combinations of precious metal prices (e.g. Escribano and Granger, 1998, Ciner, 2001).

The objective of this study is to report the long run price dynamics of the bivariate relationship between gold and silver. First, we investigate the spread, measured as the price difference between gold and silver trading as a futures contract. This is novel since previous analyses tend to focus on the price of individual series. Then, we extend the work of Ciner (2001) and Figuerola-Ferretti and Gonzalo (2010), who use the cointegration techniques of Johansen (1991) and Escribano and Granger (1998) to identify the presence of a long-term equilibrium in the gold and silver markets, by investigating the presence of long term dependence –or long memory- effects in the time-series of the gold-silver spreads.

The investigation and subsequent identification of complex dynamics, including long memory processes, in various financial time series has provoked debate in the empirical finance literature (e.g. Mandelbrot, 2001; Bianchi and Pianese, 2007), especially in terms of how the presence of these processes affect asset market efficiency (e.g. Eom, Choi and Jung, 2008; McCauley, Bassler and Gunaratne, 2008) and asset pricing (e.g. Ellis and Hudson, 2007; Takami, Tabak and Miranda, 2008) Furthermore, a key contribution of this paper is that we are able to assess the economic implications of the identified long memory process by applying a series of trading rules to the gold-silver spread time-series. Trading rules may be used to test the efficiency of markets (e.g. Brock, Lakonishok and Le Baron, 1992), since no trading rule should consistently be able to outperform a simple buy-hold strategy in the long term.
Long memory processes are typically associated with the hyperbolic decay of the autocorrelation function and are easily measured using the classical rescaled adjusted range (RAR) technique of Hurst (1951), although other approaches, such as detrended fluctuation analysis (e.g. Wan, Wei and Wu, 2011a and 2011b analysis of the gold and oil markets respectively) may be used\(^1\). One key advantage of the RAR approach is that given a daily time series of length N, the RAR may be estimated over a rolling sample (n) to produce a series of (daily) statistics (of length N- n + 1). In our case, we set n = 22 and 66, representing 1-month and 3-months, respectively and utilise a time series of Hurst statistics of 1,682 daily observations. The RAR approach also has the benefit of providing an insight into the direction of the equilibrium reverting process since it allows for differentiation between processes that revert to their long-term mean after an information shock, such as from news announcements (Christie-David, Chaudry and Koch, 2000) and those that progressively move away from the long-term mean after each new shock.

One feature common to most financial time series is the presence of short memory effects (e.g. Lo, 1991), typically observed as short term autocorrelation, which may be associated with lingering liquidity effects in a financial market. To overcome this feature of the gold-silver spread returns, we filter the series using AutoRegressive Moving Average (ARMA) techniques prior to estimating the RAR. This approach has been used previously by Szilagyi and Batten (2007) and Batten and Hamada (2009) to prefilter residuals from foreign exchange and electricity returns before testing for fractality using the rescaled range approach. The ARMA filtering approach therefore accommodates any short-term autocorrelated innovations that may be present in the return process.

We make two main contributions in this paper: First, the gold-silver spread returns reveal time-varying non-linearities, which can be identified as fractality using the commonly applied Hurst tests. The use of the local Hurst coefficient, estimated over a rolling sample period (or window) of 22-day and 66-day periods highlights the time-varying nature of this phenomenon. Time dependence in the Hurst coefficient has been

\(^1\) Owczarczuk (forthcoming) provides an excellent discussion.
identified by a number of empirical studies investigating the dynamics of financial prices including Carbone, Castelli and Stanley (2004), Batten, Ellis and Fetherston (2008) and Grech and Pamela (2008) amongst others, and so we add to this literature. Second, we test the performance of simple trading rules based upon the Hurst coefficient and importantly, find that these rules out-perform both a simple moving-average strategy, which is typically applied to trending series by traders, and a simple buy-hold strategy. This finding adds to a developing literature that utilises the Hurst coefficient as an investment tool (e.g. Clark, 2005).

These findings are also consistent with other researchers that identify time varying long-term dependence in other financial markets (Lo, 1991; Batten, Ellis and Fetherston, 2005; Cajueiro and Tabak, 2007 and 2008; Du and Ning, 2008; Grech and Pamula, 2008), other forms of nonlineairies or complex structure in financial returns (Jiang and Zhou, 2008; Takami, Tabak and Miranda, 2008), or demonstrated the economic gains from trading strategies designed to exploit stock market inefficiencies – specifically trend following trading systems (Brock, Lakonishok and LeBaron, 1992; Bessembinder and Chan, 1998; Eom et al., 2008).

Next we briefly provide information on the gold and silver futures data used for the study. Then the results from price analysis using rescaled range analysis are reported and the trading strategy applied. The final section allows for some concluding remarks.

2. Data

Lucey and Tulley (2006) provide a detailed account of studies investigating trading in the international gold and silver markets. We investigate the price of two contracts trading on the New York Mercantile Exchange (NYMEX): the deliverable 100 troy ounce nominal COMEX gold and the deliverable 5,000 troy ounce nominal COMEX silver contracts. In an economic sense these futures contract are fully arbitrageable against gold and silver trading in a variety of other worldwide cash and futures markets. Open-outcry trading commences at 08:20h/08:25h and ends at 13:30h/13:25h (gold/silver). Trading is also available simultaneously (termed side-by-side trading) on the GLOBEX electronic
trading system available on the Chicago Mercantile Exchange (CME). Our data comprises 1,746 daily observations of the near month COMEX gold and silver contract at the start of trading from January 1999 to December 2005. This number is reduced to 1,682 once the first set of observations are used for the Hurst estimations.

We first estimate the interday returns (\(\Delta P_t\)) for the price spread (\(P_t\)) between gold (\(G_t\)) and silver (\(S_t\)), where \(P_t = G_t - S_t\). Allow \(\Delta P_t = \log (P_t) - \log (P_{t-1})\) where the interval \(t-1\rightarrow t\), is 1-day. Individual asset returns (for gold and silver) are also measured as \(\Delta G_t = \log (G_t) - \log (G_{t-1})\) and \(\Delta S_t = \log (S_t) - \log (S_{t-1})\) where the interval \(t-1\rightarrow t\), is 1-day.

\[\text{(Insert Table 1 about here)}\]

Note that to the extent the returns of the underlying assets (\(\Delta G_t\) and \(\Delta S_t\)) are themselves random processes then \(\Delta P_t = \varepsilon_t\), with \(\varepsilon_t\) being a random variable, which is expected to have a mean of zero. \(\Delta P_t\) should also be both mean stationary and uncorrelated over various time increments, which is a requirement for efficient markets in the sense of Fama (1998), although, as is well known, these features are rarely present in financial markets (McCauley et al. 2008).

The descriptive statistics of our data are reported in Table 1. Over the sample period the spread varied from a maximum of US$474.31 to a minimum of US$248.63. The spread mean was US$329.32. For the return series (Figures 1d, 1e) the mean was slightly positive in all cases (\(\Delta P_t, \Delta G_t, \Delta S_t\) equaling 0.00013, 0.00014 and 0.00013 respectively) with variation on a similar scale as shown in Figures 1d and Figure 1e. Silver was more volatile over the entire sample period than the spread, or gold, measured by the standard deviation (\(\Delta P_t, \Delta G_t, \Delta S_t\), equaling 0.0044, 0.0044 and 0.0062 respectively) and the coefficient of variation (\(\Delta P_t, \Delta G_t, \Delta S_t\), equaling 3059, 3060 and 4733 respectively). However, this was not the case when the coefficient of variation (CV) was estimated over a 22-day (equal to one calendar month) rolling window (i.e. \(CV_n = \mu_n/\sigma_n\), where \(n = 22\)). The \(CV_{22}\) for gold and silver are illustrated in Figures 1a and 1b. It is clear from these figures that silver appears more stable, with less significant peaks and troughs in the \(CV_{22}\).
of returns than gold. Such differences would likely impact upon the dynamics of the long-term equilibrium between the two metals.

(Insert Figures 1a, 1b, 1c, 1d, 1e about here)

To demonstrate the time varying nature of the relationship between gold and silver we also estimate and then plot in Figure 1c, the rolling 22-day correlation between gold and silver returns (GSρ_{22}). Over the entire sample period the correlation is high and positive (GSρ_{1746} = 0.686, p=0.000). However, estimating GSρ_{22} the range varies from a maximum of 0.9675 to a minimum of -0.1524. Nonetheless, since a positive correlation is maintained over the sample period, trading strategies based on mean reversion of the spread to its average may in fact provide profitable opportunities for market participants. This finding is consistent with widely held views in commodity markets that despite the fundamental differences between the two markets, gold and silver prices tend to move together (Lucey and Tulley, 2006) thereby offering the possibility for various trading and portfolio strategies, which exploit mean reversion in the spread returns. This possibility is considered further when the rescaled range statistic is estimated in the next section and when trading strategies based upon the local Hurst exponent are implemented in section 4.

3. Rescaled Adjusted Range Analysis

The presence of long-term dependence in the spread returns (ΔP_t) between gold and silver may be measured using statistical techniques based on the Hurst (1951) rescaled range analysis, after accommodating for any short-term autocorrelated innovations in the return process (Batten, Ellis and Fetherston, 2008). The filtering process is readily accomplished via ARMA models and forms the basis for RAR (Batten and Szilagyi, 2007; Batten and Hamada, 2009). Of specific interest is the residual ψ_t after applying various filters (AR(0) →ARMA (2,1)) to ΔP_t. Consider an ARMA(2,1) model of the form

$$\Delta P_t = \alpha_0 + \beta_1 \Delta P_{t-1} + \beta_2 \Delta P_{t-2} + X_1 \lambda \psi_{t-1} + \psi_t$$  \hspace{1cm} (1)

which systematic analysis is found to provide the best fit to the data with $\beta_1 = 0.4802$ (p=0.06), $\beta_2 = 0.1109$ (p =0.000) and $X_1 = 0.5780$ (p = 0.024).
For each $\psi_t$ over the subsample $n$, the classical rescaled adjusted range $(R/\sigma)_n$ of Hurst (1951) and Mandelbrot and Wallis (1969) is calculated as:

$$
(R/\sigma)_n = \frac{1}{\sigma_n} \left[ \max_{1 \leq k \leq n} \sum_{j=1}^{k} (\psi_j - \mu_n) - \min_{1 \leq k \leq n} \sum_{j=1}^{k} (\psi_j - \mu_n) \right]
$$

(2)

where $\mu_n$ is the mean and $\sigma_n$ is the standard deviation of $\psi_t$ over an overlapping sample of length $n$

$$
\sigma_n = \left[ \frac{1}{n} \sum_{j=1}^{n} (\psi_j - \mu_n)^2 \right]^{0.5}
$$

(3)

In order to capture the time-varying nature of dependence in $\psi_t$, this study employs a local measure of the Hurst exponent ($h$) as used by Szilagyi and Batten (2007) and Batten and Hamada (2009). The local version ($h$) of the Hurst exponent is then estimated for $(N-n+1)$ times overlapping subseries of these various length $n$:

$$
h_n = \frac{\log(R/\sigma)_n}{\log n}
$$

(4)

where $n$ is set to either 22 days or 66 days, which is equivalent to a standard one and three month period. This procedure in effect creates a time-series of exponent values, the change in whose value can be measured over time. The averages of the local Hurst ($h$) for the entire sample period are summarised in Table 2. The top row in this table records the filtering technique applied to $\psi_t$. These four techniques range from AR(0) – no filtering - to ARMA (2,1) as per Equation 1.

(Insert Table 2 about here)

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2 The filtering using the ARMA (2,1) affects the residuals of equation 1. It is the residual series $\psi_t$ that is used to estimate the $(R/\sigma)_n$ of Hurst (1951) using $(R/\sigma)_n = 1/\sigma_n$. The transformation shown in Equation (4) is then applied to estimate the local Hurst ($h$) over a 22-day and 66-day rolling window.
Recall from Hurst (1951) that under the null hypothesis of no long-term dependence, the value of $h_n = 0.5$ (a white noise process). For time-series exhibiting positive long-term dependence, the observed value of the exponent $h_n > 0.5$. Time-series containing negative dependence are mean-reverting and alternatively characterised by $h_n < 0.5$ (termed pink noise by Mulligan, 2004). Note that while the original sample contains 1,746 observations, estimation of the 22-day (66-day) local Hurst coefficient is based on the returns from $R_0 \rightarrow R_{t-22}$ (and $R_0 \rightarrow R_{t-66}$) observations. To allow convergence to a stable Hurst coefficient we ignore the first 64 estimations from the original sample.

From Table 1, the mean for $h_{22}$ varies from (0.7070 for AR(0) to 0.7170 (ARMA(2,1)). The scores for $h_{66}$ are lower, suggesting the series is becoming more random as the sample length $n$ in Equation (4) increases, and vary from (0.6487 for AR(0) to 0.6496 (ARMA(2,1)). Note too, that these different filters have little effect on the size of the $h$-statistic as evidenced by the overlapping of the confidence intervals. This is due to the fact that the long memory, by definition, relates to the lingering effects of hyperbolic decay in the autocorrelation of the return series. One economic explanation for this property is due to liquidity effects in the market. Even, allowing for a 95% confidence interval these local Hurst coefficients are consistent with positive long term dependence. For positively dependent processes another price movement further away from the mean (or long term equilibrium) will follow the earlier movement away from equilibrium. Thus, they are trend-reinforcing processes (Mulligan, 2004), such that the spread should tend to get larger, or smaller, depending on whether the previous change in price was positive, or negative.

(Insert Figure 2a and 2b about here)

Nonetheless, a plot of $h_{22}$ and $h_{66}$ over the entire sample period (shown in Figures 2a and 2b) shows considerable variation in the statistic and rare episodes when the statistic was below 0.5000. In the case of $h_{22}$ the minimum value was 0.3296 and the maximum was
1.1667 while for $h_{66}$ the minimum value was 0.4071 and the maximum was 0.9399. Values below 0.5000 are consistent with negative dependence where a price movement towards the equilibrium should follow a movement away from equilibrium.

(Insert Table 3 about here, Figure 3 about here)

4. Trading Implications

To investigate the trading implications we first divide the unfiltered return series ($\Delta P_t$) into varying holding periods (HP) from 1-day to 22-days from the start day return at $t_0$ (i.e. $R_0$) These holding period returns are reported in Table 3 and reflect the reward to the investor for a buy-hold investment strategy for these specified number of days. Note that kurtosis and skewness declines with holding period length, while the mean return increases as days are added to the holding period. However, the return over the holding period on an average daily basis is not linear. This is clearer in Figure 3, which plots the average daily return over the varying holding period lengths (reported in the third column of Table 3). Figure 3 clearly shows that there is an advantage to holding a portfolio for the next 2 days instead of just one day, but this advantage declines quickly for the next 6 days, where after it increases again steadily for the next period. The optimal holding period (that is the holding period with the largest average daily return) is a period of 18-days from today.

(Insert Table 4 about here)

A number of trading strategies were then applied to the return series. The first of these is one commonly applied to trend-following systems: a moving average. Brock, Lakonishoko and Le Baron (1992), in a leading paper, demonstrated the success of moving average trading systems over others. While moving averages of varying length

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3 Note this high level of the Hurst exponent (greater than 1) appears to be a statistical artefact due to extreme volatility in the daily gold-silver spread price which moved from US$250 to US$320 in the period from 9/7/1999 to 11/3/1999 (nearly a 30% move). Removing the spreads from the period 10/1/1999 to 10/18/1999 eliminates this anomaly. Alternately, if the estimation sample length is increased to 66, then the anomaly dissipates. Removing these extreme observations does affect the estimated average Hurst value. For example, in Table 2 the ARMA (2,1) 22-day Hurst is now 0.7048 versus the reported number of 0.7170 and for the ARMA (2,1) 66-day Hurst is 0.6475 versus the reported number of 0.6496, which just lie within the confidence interval.
could be considered for the sake of brevity we investigate a moving average of length 22, which matches the other estimations for the local Hurst coefficient.

The first rule that is applied is to buy when the 22-day moving average price is greater than the price today (ie \( P_t > MA_{22} \) in Table 4). If this rule is triggered, the subsequent returns to the next day (\( HP = R_0 \rightarrow R_1 \)), the next 5-days (\( HP = R_0 \rightarrow R_5 \)), the next 10-days (\( HP = R_0 \rightarrow R_{10} \)) and the next 20-days (\( HP = R_0 \rightarrow R_{20} \)) are reported in the first four rows of Table 4. The frequency of this rule is reported as \( N=0 \) (reject) and \( N=1 \) (accept). In all cases there were more accepted applications of the rule than rejected. However, while mean returns from the accepted application of the rule (\( N=1 \)) steadily increased from 0.00133 for the next day to 0.00252 for the next 20-days, an \( F \)-test of difference in the means shows that only the next one-day and 5-day holding periods were statistically different from the alternate (rejected) portfolio returns (\( N=0 \)). Importantly, the returns for the return on the average buy-hold portfolio held for 20-days (reported in Table 3 as 0.003079), is higher than the return for an accepted application of the rule for 20-days (Table 4, \( N=1 \), mean of 0.00252). Thus, while the moving average rule offers potential short-term gains these appear to diminish as the holding period is lengthened. This short-term gain is consistent with trading models that exploit the autoregressive features of the series. These were previously identified in the ARMA model (equation 1), which identifies positive one and two day autocorrelation in the return series.

The next set of rules that are applied utilise the time-series of local Hurst coefficients, whose averages are reported in Table 2. For the sake of brevity only the unfiltered local Hurst (AR0) estimated over 22-days is reported although similar findings apply for the Hurst (AR0) 66-day estimations. There are two conditions considered to provide the perspective when the return time series is mean reverting (\( H_{22} < 0.4 \); 27 instances when the rule is accepted), or trend-reinforcing (\( H_{22} > 0.6 \); 1342 instances when the rule is accepted). The same holding period lengths for the moving average rule are applied (next day, next 5-days, next 10-days and next 20-days). Apart from one exception (for \( H_{22} < 0.4 \), for the next day holding period), which is the likely consequence of the positive autoregressive structure mentioned earlier, the \( F \)-test of difference in the other mean
returns shows significant differences in returns over the alternate portfolio. Also, these returns are significantly different to the returns from the average buy-hold portfolios of varying holding periods reported in Table 3. For example, from Table 4, for the Hurst rule of $H_{22} > 0.6$, for the 20-day holding period, the mean return is 0.00461 compared with the 20-day buy-hold of 0.003079 (Table 2). Note that to effect a profitable trade the investor is required to sell when $H_{22} < 0.4$ (a bet on mean reversion), and buy when $H_{22} > 0.6$ (a bet on trend-reinforcement).

5. Conclusions
This study investigates the relationship between gold and silver trading as a futures contract on COMEX from January 1999 to December 2005. During this period the correlation between gold and silver returns was positive and high even though the relationship itself was unstable. We apply techniques from fractal geometry after accommodating underlying autoregressive behaviour to investigate the long term dynamics of the spread between these two contracts. Using a local Hurst exponent we find episodes of both positive and negative dependence, though the positive dependent relationship appears to be dominant. This last finding is suggestive of a time-varying fractal structure in the spread returns. Positive dependence (consistent with a Hurst coefficient $> 0.5$) in the gold-silver spread returns suggests the series will not immediately revert to its average or long term mean, thereby offering traders limited profit opportunities. To test this proposition we test a number of simple trading rules based upon the Hurst coefficient. We find that trading rules requiring the investor to sell when the local Hurst coefficient, estimated over a 22-day window, is less than 0.4 (a bet on mean reversion), and buying when the local Hurst coefficient, estimated over a 22-day window, is greater than 0.6 (a bet on trend-reinforcement) out-performs a simple buy-hold and moving-average strategy. This result adds to a growing body of work which finds significant arbitrage possibilities remaining in financial markets despite the advent of improved pricing and information technology (Avellaneda and Lee, 2010)
References


Clark, Andrew (2005): The use of Hurst and effective return in investing, Quantitative Finance, 5:1, 1-8


Figure 1a: The coefficient of variation of daily silver returns estimated on a 22-day rolling window.

Figure 1b: The coefficient of variation of daily gold returns estimated on a 22-day rolling window.
Figure 1c: The correlation between daily gold and silver returns estimated on a 22-day rolling window.

Figure 1d: The average 22-day daily gold and silver returns estimated on a rolling window.
Figure 1e: The average 22-day gold-silver spread returns estimated on a rolling window
Figure 2a: The local Hurst estimated using an ARMA(2,1) filter on the spread return between daily gold and silver (estimated on a 22-day rolling window).

Figure 2b: The local Hurst estimated using an ARMA(2,1) filter on the spread return between daily gold and silver (estimated on a 66-day rolling window).
Figure 3: Plot of average daily gold-silver spread returns for holding periods from one day to one month
Table 1: Descriptive statistics of the gold-silver spread
(Daily spreads from January 1999 to December 2005)

<table>
<thead>
<tr>
<th></th>
<th>Gold-silver spread ( P_t = G_t - S_t )</th>
<th>Gold-silver spread return ( \Delta P_t = \log(P_t) - \log(P_{t-1}) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean</td>
<td>329.32</td>
<td>0.00013</td>
</tr>
<tr>
<td>Median</td>
<td>309.44</td>
<td>0.00015</td>
</tr>
<tr>
<td>Maximum</td>
<td>474.31</td>
<td>0.04464</td>
</tr>
<tr>
<td>Minimum</td>
<td>248.63</td>
<td>-0.02873</td>
</tr>
<tr>
<td>Standard Deviation</td>
<td>62.55</td>
<td>0.00440</td>
</tr>
<tr>
<td>Skewness</td>
<td>0.505</td>
<td>0.902</td>
</tr>
<tr>
<td>Kurtosis</td>
<td>1.801</td>
<td>16.411</td>
</tr>
<tr>
<td>Jarque-Bera</td>
<td>172.34</td>
<td>12839.13</td>
</tr>
<tr>
<td>Probability</td>
<td>0.000</td>
<td>0.00000</td>
</tr>
<tr>
<td>Observations</td>
<td>1,682</td>
<td>1,682</td>
</tr>
</tbody>
</table>

Note: The original sample contains 1,746 observations. The later 22-day (66-day) estimation of the Hurst coefficient is based on the returns from \( R_0 \rightarrow R_{t-22} \) (and \( R_0 \rightarrow R_{t-66} \)) observations. To allow convergence to a stable Hurst coefficient we simply report the subsequent 1682 observations.
Table 2: Local Hurst exponents estimated using 22 and 66-day rolling windows

<table>
<thead>
<tr>
<th>Overall Sample</th>
<th>22-day rolling window (1 month)</th>
<th>66-day rolling window (3 months)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>AR(0)</td>
<td>AR(1)</td>
</tr>
<tr>
<td>mean</td>
<td>0.7071</td>
<td>0.7370</td>
</tr>
<tr>
<td>Standard deviation</td>
<td>0.1314</td>
<td>0.1328</td>
</tr>
<tr>
<td>95% Confidence interval</td>
<td>0.7007-0.7133</td>
<td>0.7239-0.7366</td>
</tr>
</tbody>
</table>

Table 3: Returns over different holding periods (1-day to 22-days)

<table>
<thead>
<tr>
<th>Return (R) Window</th>
<th>Mean</th>
<th>Average Daily</th>
<th>Standard Deviation</th>
<th>Kurtosis</th>
<th>Skewness</th>
</tr>
</thead>
<tbody>
<tr>
<td>R₀</td>
<td>0.0001443</td>
<td>0.000144349</td>
<td>0.004417358</td>
<td>0.8478223</td>
<td>12.91632</td>
</tr>
<tr>
<td>R₀→R₁</td>
<td>0.0002910</td>
<td>0.000145520</td>
<td>0.005920985</td>
<td>1.0070222</td>
<td>8.732371</td>
</tr>
<tr>
<td>R₀→R₂</td>
<td>0.0004361</td>
<td>0.000145371</td>
<td>0.008408646</td>
<td>0.9301216</td>
<td>6.773518</td>
</tr>
<tr>
<td>R₀→R₃</td>
<td>0.0007179</td>
<td>0.000143574</td>
<td>0.009547577</td>
<td>1.0231929</td>
<td>6.256877</td>
</tr>
<tr>
<td>R₀→R₄</td>
<td>0.0008565</td>
<td>0.000142749</td>
<td>0.010560701</td>
<td>1.1241108</td>
<td>5.873518</td>
</tr>
<tr>
<td>R₀→R₅</td>
<td>0.0009934</td>
<td>0.000141918</td>
<td>0.011531883</td>
<td>1.1706315</td>
<td>5.118051</td>
</tr>
<tr>
<td>R₀→R₆</td>
<td>0.0011374</td>
<td>0.000142175</td>
<td>0.012337697</td>
<td>1.1952264</td>
<td>5.778539</td>
</tr>
<tr>
<td>R₀→R₇</td>
<td>0.0012822</td>
<td>0.000142462</td>
<td>0.013090507</td>
<td>1.1863138</td>
<td>5.237252</td>
</tr>
<tr>
<td>R₀→R₈</td>
<td>0.0014258</td>
<td>0.000142576</td>
<td>0.013770619</td>
<td>1.2489172</td>
<td>4.704415</td>
</tr>
<tr>
<td>R₀→R₉</td>
<td>0.0015711</td>
<td>0.000142826</td>
<td>0.014435979</td>
<td>1.2597065</td>
<td>4.978512</td>
</tr>
<tr>
<td>R₀→R₁₀</td>
<td>0.0017207</td>
<td>0.000143395</td>
<td>0.015047962</td>
<td>1.2411078</td>
<td>4.672399</td>
</tr>
<tr>
<td>R₀→R₁₁</td>
<td>0.0018772</td>
<td>0.000144402</td>
<td>0.015588680</td>
<td>1.2269747</td>
<td>4.620100</td>
</tr>
<tr>
<td>R₀→R₁₂</td>
<td>0.0020384</td>
<td>0.000145601</td>
<td>0.016069123</td>
<td>1.2125824</td>
<td>6.145907</td>
</tr>
<tr>
<td>R₀→R₁₃</td>
<td>0.0021926</td>
<td>0.000146175</td>
<td>0.016539234</td>
<td>1.2111407</td>
<td>6.949431</td>
</tr>
<tr>
<td>R₀→R₁₄</td>
<td>0.0023446</td>
<td>0.000146540</td>
<td>0.016999905</td>
<td>1.1954588</td>
<td>5.122018</td>
</tr>
<tr>
<td>R₀→R₁₅</td>
<td>0.0024945</td>
<td>0.000146736</td>
<td>0.017448983</td>
<td>1.1795708</td>
<td>5.321579</td>
</tr>
<tr>
<td>R₀→R₁₆</td>
<td>0.0026419</td>
<td>0.000146773</td>
<td>0.017885264</td>
<td>1.1508532</td>
<td>4.945140</td>
</tr>
<tr>
<td>R₀→R₁₇</td>
<td>0.0028879</td>
<td>0.000146714</td>
<td>0.018343986</td>
<td>1.0978180</td>
<td>4.601016</td>
</tr>
<tr>
<td>R₀→R₁₈</td>
<td>0.0029360</td>
<td>0.000146802</td>
<td>0.018774185</td>
<td>1.0558345</td>
<td>4.244551</td>
</tr>
<tr>
<td>R₀→R₁₉</td>
<td>0.0030790</td>
<td>0.000146621</td>
<td>0.019170964</td>
<td>1.0150762</td>
<td>3.913951</td>
</tr>
<tr>
<td>R₀→R₂₀</td>
<td>0.0032216</td>
<td>0.000146438</td>
<td>0.019545214</td>
<td>0.9705729</td>
<td>3.697957</td>
</tr>
<tr>
<td>R₀→R₂₁</td>
<td>0.0033633</td>
<td>0.000146233</td>
<td>0.019907824</td>
<td>0.9214373</td>
<td>3.398959</td>
</tr>
</tbody>
</table>
Table 4: Analysis of variance (ANOVA) of differences in mean returns based on various trading rules over different holding periods

<table>
<thead>
<tr>
<th>Trading Rule</th>
<th>N=0</th>
<th>N=1</th>
<th>Mean N=0</th>
<th>Mean N=1</th>
<th>Standard Deviation N=0</th>
<th>Standard Deviation N=1</th>
<th>F-test</th>
<th>p-value</th>
<th>Adjusted R²</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pt &gt; MA22 HP = R₀→R₁</td>
<td>812</td>
<td>889</td>
<td>-0.00085</td>
<td>0.00133</td>
<td>0.004834</td>
<td>0.00665</td>
<td>58.59</td>
<td>0.000</td>
<td>3.28%</td>
</tr>
<tr>
<td>Pt &gt; MA22 HP = R₀→R₅</td>
<td>810</td>
<td>887</td>
<td>-0.00001</td>
<td>0.00164</td>
<td>0.00846</td>
<td>0.01221</td>
<td>10.18</td>
<td>0.001</td>
<td>0.54%</td>
</tr>
<tr>
<td>Pt &gt; MA22 HP = R₀→R₁₅</td>
<td>808</td>
<td>884</td>
<td>0.00108</td>
<td>0.00206</td>
<td>0.01265</td>
<td>0.01603</td>
<td>1.96</td>
<td>0.162</td>
<td>0.06%</td>
</tr>
<tr>
<td>Pt &gt; MA22 HP = R₀→R₂₀</td>
<td>808</td>
<td>874</td>
<td>0.00387</td>
<td>0.00252</td>
<td>0.01979</td>
<td>0.01876</td>
<td>2.08</td>
<td>0.150</td>
<td>0.06%</td>
</tr>
<tr>
<td>Pt, H₂₂ &lt; 0.4 HP = R₀→R₁</td>
<td>1,674</td>
<td>27</td>
<td>0.00331</td>
<td>-0.00073</td>
<td>0.00591</td>
<td>0.00829</td>
<td>0.81</td>
<td>0.368</td>
<td>0.00%</td>
</tr>
<tr>
<td>Pt, H₂₂ &lt; 0.4 HP = R₀→R₅</td>
<td>1,670</td>
<td>27</td>
<td>0.00098</td>
<td>-0.00681</td>
<td>0.01061</td>
<td>0.00775</td>
<td>14.40</td>
<td>0.000</td>
<td>0.78%</td>
</tr>
<tr>
<td>Pt, H₂₂ &lt; 0.4 HP = R₀→R₁₅</td>
<td>1,665</td>
<td>27</td>
<td>0.00181</td>
<td>-0.01184</td>
<td>0.01449</td>
<td>0.00836</td>
<td>23.81</td>
<td>0.000</td>
<td>1.33%</td>
</tr>
<tr>
<td>Pt, H₂₂ &lt; 0.4 HP = R₀→R₂₀</td>
<td>1,655</td>
<td>27</td>
<td>0.00345</td>
<td>-0.01425</td>
<td>0.01925</td>
<td>0.01072</td>
<td>22.71</td>
<td>0.000</td>
<td>1.28%</td>
</tr>
<tr>
<td>Pt, H₂₂ &gt; 0.6 HP = R₀→R₁</td>
<td>359</td>
<td>1,342</td>
<td>-0.00021</td>
<td>0.00042</td>
<td>0.00621</td>
<td>0.00588</td>
<td>3.14</td>
<td>0.076</td>
<td>0.13%</td>
</tr>
<tr>
<td>Pt, H₂₂ &gt; 0.6 HP = R₀→R₅</td>
<td>359</td>
<td>1,338</td>
<td>-0.00071</td>
<td>0.00127</td>
<td>0.01166</td>
<td>0.01028</td>
<td>9.96</td>
<td>0.002</td>
<td>0.53%</td>
</tr>
<tr>
<td>Pt, H₂₂ &gt; 0.6 HP = R₀→R₁₅</td>
<td>359</td>
<td>1,333</td>
<td>-0.00128</td>
<td>0.00237</td>
<td>0.01442</td>
<td>0.01445</td>
<td>18.05</td>
<td>0.000</td>
<td>1.00%</td>
</tr>
<tr>
<td>Pt, H₂₂ &gt; 0.6 HP = R₀→R₂₀</td>
<td>359</td>
<td>1,323</td>
<td>-0.00214</td>
<td>0.00461</td>
<td>0.01727</td>
<td>0.01953</td>
<td>35.36</td>
<td>0.000</td>
<td>2.00%</td>
</tr>
</tbody>
</table>
Note: The original sample contains 1,746 observations. The 22-day Hurst coefficient is based on the previous 22 observations, which allows 1746-22 Hurst (H) estimations. To allow convergence to a stable Hurst coefficient we ignore the next 22 observations. The holding period (HP) return (R) varies from the return to the next day (HP = R_0\rightarrow R_1) to the next 20-days (HP = R_0\rightarrow R_{20}). Therefore, the sum of N= 0 and N= 1, varies from 1,701 (for a one-day holding period) to 1,682 (for a 20-day holding period). The trading rules are buy when the price at t=0 > previous 22-day Moving Average (Pt > MA22), sell the price at t=0 when the previous 22-day Hurst coefficient is < 0.4 (Pt, H_{22} < 0.4), and buy the price at t=0 when the previous 22-day Hurst coefficient is > 0.6 (Pt, H_{22} > 0.6).