



Do bubbles occur in the gold price? An investigation of gold lease rates and Markov Switching models

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Abstract

We assess whether two classes of bubbles occur in the spot price of gold, rational speculative and periodically bursting bubbles, using gold's lease rates for the first time in the literature as a measure of its fundamental value. This question is of particular significance as these are the only observable market measures of a yield that can be earned from gold. We use unit root and cointegration tests to look for rational speculative bubbles and Markov Switching Augmented Dickey–Fuller tests for periodically bursting bubbles.

ADF and cointegration tests point to a rational speculative bubble. The more theoretically valid Markov Switching ADF test gives mixed evidence. No bubble is found to be present if we allow the variance to switch between regimes, the gold and its lease rate relationship is instead characterised by high and low variance periods. Imposing a constant variance gives evidence of a bubble for the 2, 3 and 12 month lease rates, but no bubble when we use the 1 and 6 month rates as determinants.

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1. Introduction

In recent years gold has enjoyed a renewed prominence as a financial asset and is now being purchased once again in increasing quantities by a wide variety of sectors. Most major

central banks up to 2010 had a long term policy of selling gold but in 2010 and 2011 they purchased 77 tonnes and 455 tonnes respectively. New investment vehicles have emerged to allow small investors to buy gold with Exchange Traded Funds (ETFs) buying 368 and 162 tonnes of gold in 2010 and 2011 respectively (GFMS Gold Survey, 2012) though they have been reducing their holdings in 2013. This has happened to a backdrop of huge increases in the price of gold in recent years, rising from just under \$300 per fine troy ounce in 2000 to just over \$1900 in mid-2011 before a sharp price fall in 2013 down to approximately \$1200. Fig. 1 below shows the rapidly increasing volume of open interest in gold futures contracts since 2000 (Fig. 2).

With both investors and the official sector now increasingly investing in this asset class again, new nominal highs in the gold price being reached and followed by large price falls, it

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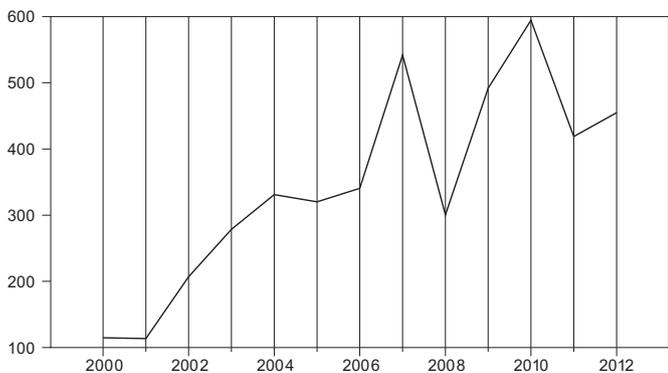


Fig. 1. Open interest in gold futures contracts on COMEX. Source: CFTC, Commitment of traders.

seems appropriate that an attempt is made to assess whether gold’s price is justified by a market based fundamental determinant, that is, by the income that can be earned by owning it.

We use gold lease rates, the interest that can be earned by lending physical gold at various maturities, as a way to measure gold’s true value. Despite these observable market measures of a cash flow that can be earned from gold existing since 1989, lease rates have never been used to assess whether bubbles occur in gold. Previous papers in this area have used convenience yield and real interest rates. We argue that lease rates are a superior way examine whether bubbles occur in gold prices for a number of reasons.

We test for rational speculative bubbles by testing for the existence of stationarity and cointegrating relationships between the spot price of gold and its lease rates. We also apply more advanced and theoretically valid tests for periodically bursting bubbles using Markov-Switching Augmented Dickey–Fuller (MSADF) tests.

The rest of the paper is organised as follows. Section 2 discusses the literature on the fundamental and macroeconomic drivers of the price of gold. Section 3 gives an overview of the literature around bubbles, models of bubbles in the price

of gold and evidence on bubbles in the price of gold. Section 4 discusses the methods used to answer the question, Section 5 describes the data used and Section 6 presents results while the conclusions are in Section 7.

2. What drives the price of gold?

2.1. Fundamental microeconomic drivers

For any asset we can consider a fundamental microeconomic driver of its value to be the earnable return it can provide to its owner. All things being equal, a higher return should increase the asset’s value from a supply and demand perspective. In the case of gold, a commodity, this causes some difficulty. Prominent investors such as Warren Buffet assert that gold has no such earnable return and as such is, in the limit, valueless (Denning, 2012). This is not quite the case. There are at least two measures of such a return; the convenience yield and gold lease rates.

Previously gold’s convenience yield has been used to measure its fundamental value in studies such as Pindyck (1993), Went, Jirasakuldech, and Emekter (2009) and Bialkowski, Bohl, Stephan, and Wisniewski (2011). A commodity’s convenience yield is the benefit the holder of the physical commodity receives relative to the owner of a futures or forward contract on the asset and reflects the markets view about its future supply (Hull, 2006). It is measured as in equation (2.1) below:

$$F_t = P_t \cdot e^{(r_f - CY_t)t} \tag{2.1}$$

Where F_t is a futures or forward price at time t , P_t is the spot price at time t , r_f is the risk free rate of interest and CY_t is the convenience yield at time t . P_t can also be replaced with a futures or forward price of a different maturity to F_t .

Convenience yield is then the rate that allows for no arbitrage, as the futures price is equal to the spot price adjusted for the opportunity cost of investing in physical gold. This yield

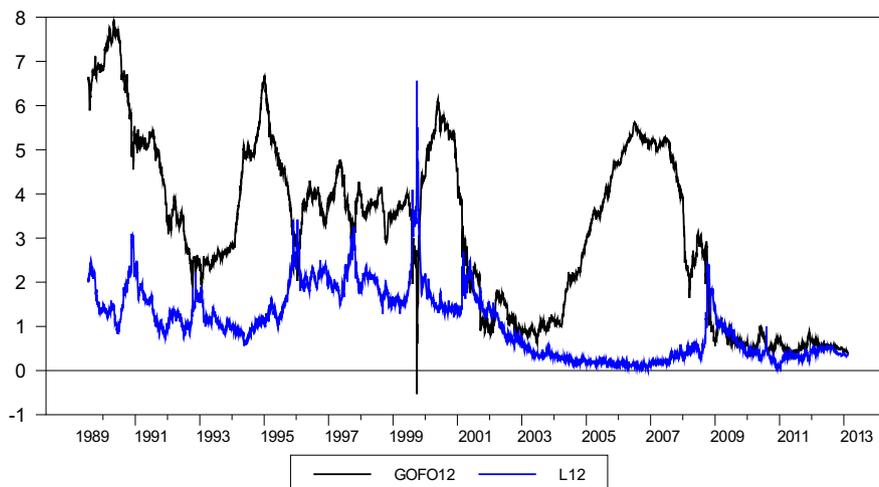


Fig. 2. 12 month GOFO and gold lease rate. Source: LBMA website.

can then be used in a similar way to the yield on a financial asset such as a bond or a share to compute its present value. It can be thought of intuitively as the indirect benefits the holder of a commodity receives such as ease of access for production, preventing hold up problems and, from a financial asset perspective for gold, having the metal in your portfolio as a hedge against market risk or safe haven in a financial crisis as discussed in [Baur and Lucey \(2010\)](#).

Another, and arguably better, measure of a return that can be earned by owning gold are gold lease rates. Gold lease rates have not previously been used to assess gold's value from the perspective of fundamental microeconomic drivers. Gold lease rates are the annualised over the counter interest rates that can be earned by lending gold over 1, 2, 3, 6 or 12 months. They are technically referred to as derived lease rates, which is calculated daily as the London Interbank Offer Rate (LIBOR) at a given maturity minus the Gold Offer Forward Rate (GOFO) at that maturity ([LBMA, 2008](#)). This can be used by an owner of gold to earn a cashflow similar to the dividends or coupon payments that can be earned by owning equities or bonds.

It is derived in theory as follows: A lends US Dollars to B at LIBOR and B lends gold to A at GOFO. At the end of the period both parties return the amount of gold or dollars borrowed plus the interest agreed. The difference is the lease rate ([LBMA, 2008](#)). As GOFO is generally lower than LIBOR dollars can usually be borrowed more cheaply in this way. From this we see that the lease rate can be viewed as the market's valuation of gold's worth as collateral for a US Dollar loan. In reality offsetting loans are not necessary; gold is simply lent and borrowed at the lease rate. The evolution of the 12 month GOFO and gold lease rates are shown below.

The main borrowers of gold have been mining companies and jewellers who would borrow a portion of their expected mine or jewellery output in the lease market, and sell it to finance production costs. This provided a cheap source of dollar finance and also a natural hedge against gold prices as the leased gold would be returned from their mine production in ounces of gold or from the sale of jewellery at current market prices, making gold price changes irrelevant. Hedging by gold miners fell dramatically in the early 21st century as gold prices rose steadily but in 2011 net producer hedging began again ([GFMS Gold Survey, 2012](#)).

The payment made at the end of the lease is calculated as:

$$\text{Lease Rate}_m * \text{Ounces of Gold} * \text{PM Fixing} * \frac{m}{360} \quad (2.2)$$

Where m is the number of days to maturity and assuming 30 days per month ([LBMA, 2008](#)).

This paper posits that gold lease rates are superior to the convenience yield as a measure of gold's microeconomic fundamental value for a number of reasons, both major and minor.

Lease rates are a directly observable cashflow that can be earned from owning gold, whereas convenience yield must be inferred from the difference between the spot and futures

prices of a commodity, or two futures prices of different maturities. This raises the problem that as a measure of gold's value convenience yield is derived from two gold prices, the variables that it seeks to explain (see equation (2.1) above).

The concept of convenience yield is primarily intended for consumption commodities as it measures the benefits of having easy access to a commodity to allow smooth production and being able to avoid hold up problems. Investment assets are assumed to have a zero convenience yield; if they do not arbitrage opportunities present themselves for investors ([Hull, 2006](#)). While gold is consumed in electronics and dentistry, it is primarily held for investment with annual jewellery and pure investment demand accounting for over 80% of annual demand between 2000 and 2011 ([GFMS Gold Survey, 2012](#)). This leaves it in a slightly grey area where the convenience yield applies, but not perfectly.

Gold over the counter market rates, such as lease rates and GOFO, are rarely discussed in the literature. [Levin, Abhyankar, and Ghosh \(1994\)](#) provide an arbitrage model arguing that the lease rate is a proxy for real interest rates. [Levin and Wright \(2006\)](#) use this finding to argue that the lease rate, as a proxy for the real interest rate, is the opportunity cost of holding gold as this is the amount that could have been earned in a risk free investment in their model of the gold price. [Lucey and O'Connor \(2013\)](#) look at GOFO's ability to forecast future spot prices and find behaviour factors explain some its failure to do so.

[Barone-Adesi, Geman, and Theal \(2010\)](#) find that lease rates are a good measure of the convenience yield of owning gold. This fits with the theory put forward by [Levin and Wright \(2006\)](#) where the lease rate is composed of the convenience yield of gold as well as default risk, given that gold lease rates are an over the counter transaction and subject to the risk that one party may default.

Overall, there is a limited amount of analysis of the fundamental microeconomic drivers of gold. As a consequence, there have been few papers that have examined bubbles in the traditional manner, of consistent deviations from fundamentally justified levels.

2.2. Macroeconomic drivers

As with any other financial asset, the greater macroeconomy influences gold prices and this area is where the majority of research on the gold market is concentrated. We find in the literature that gold relates to other macroeconomic variables in predictable and economically sensible ways. Thus [Levin and Wright \(2006\)](#) find that US inflation is the sole correlate of the gold price over the long term. They argue that the relationship between gold and US CPI is an artefact of the cost of gold production.

[Christie-David, Chaudhry, and Koch \(2000\)](#) use intraday data to assess whether macroeconomics news affects the price of gold futures. Consumer Price Index (CPI) releases were found to have a strong effect on returns, which fits with other research that finds that gold can be a hedge against inflation such as [Ghosh, Levin, Macmillan, and Wright \(2004\)](#). This is

posited to be because gold is a currency whose value cannot be diminished by increasing supply through printing, as is the case of fiat currencies such as the dollar or the euro, and provides an alternative reason to Levin and Wright (2006). Kutan and Aksoy (2004) test for the effect of news on the Turkish Gold Market and find that Turkish CPI does not affect the US Dollar gold price, further backing up Levin and Wright's assertion that US CPI and not world prices are what matters.

Gold is traded primarily in dollars and the strength of the dollar (as measured by the trade weighted exchange rate) is found to be a strong short term determinant by Levin and Wright (2006) and Kaufman and Winters (1989). A strong dollar makes gold cheaper for other nations to purchase and increases their demand. This then drives up the price of gold explaining their negative relationship, as is found also by Tully and Lucey (2007) and Sari, Hammoudeh, and Soytas (2010). However O'Connor and Lucey (2012) show that the trade weighted value of a number of currencies have negative relationships with the price of gold expressed in that currency. It implies that when a currency is on average losing value against all other currencies, it is also likely to be losing value against gold. This indicates that negative correlation between gold and the trade weighted value of the dollar may be a spurious one and points to currency like qualities in gold.

Interest rates also figure as an important explanatory variable. Koutsoyiannis (1983) find a strong link to nominal US interest rates and Diba and Grossman (1984) find a link to real interest rates in the US. The underlying economic theory points to the fact that the opportunity cost of holding gold is the interest that could have been earned from holding another currency on deposit. Lawrence (2003) argues against these points and using quarterly data from 1979 to 2001 finds that there is no statistically significant link between gold and these macroeconomic variables.

Baur and Lucey (2010) examine gold's relationship market crashes, its safe haven property. They study the relationship between U.S., U.K. and German stock and bond returns and gold returns. They find that gold is a hedge and a safe haven for stocks. However gold only acts as a safe haven for 15 days after a market crash. Baur and McDermott (2010) extend this analysis to a more international sample with similar results. Coudert and Feingold (2011) find a negative or null correlation between gold and a number of major stock markets indexes.

Deciding which of these factors represent true macroeconomic drivers and which are spurious or speculative factors represents a real issue, as mentioned above and in Baur and Glover (2012). Focusing on assessing gold's true value from a microeconomic perspective rather than from the macroeconomic standpoint allows us to use an undisputed measure of the benefit of holding gold, the income that can be earned from it.

3. Rational speculative bubbles in asset prices

What is a bubble? In common parlance we are aware that it means an asset price which is "too high", relative to some fundamental driver, and which must inevitably burst. More

formally, Gurkaynak (2008:166) defines a rational speculative bubble (for equities) as being when "investors are willing to pay more for the stock than they know is justified by the value of the discounted dividend stream." They do this in expectation of being able to sell at a price in the future above the present value of discounted dividends, making the high price an equilibrium price. Irrational bubbles in asset prices, where investors believe the market to be overvalued but do not go short focus on the difference between investor actions and beliefs are used in studies such as Vissing-Jorgensen (2003) but are outside the scope of this research.

Gurkaynak (2008) shows that for a normal asset with an observable yield it's fundamental, no arbitrage, value is equal to the discounted stream of future cash receipts or:

$$P_t = \sum_{i=1}^{\infty} \frac{E_t(P_{t+i} + C_{t+i})}{(1+r)^i} \quad (3.1)$$

Where P_t is the value of the asset at time t , C_{t+i} is the cash flow derived from owning the asset earned at time $t+i$ and r is the risk free rate of interest.

If a rational bubble exists, then the value of the asset is made up of two components: the fundamental market value, the discounted value of expected future cash flows, as given by equation (3.1) and a bubble term, B_t . The true value of the asset is then given by equation (3.2):

$$P_t = \sum_{i=1}^{\infty} \frac{E_t(C_{t+i})}{(1+r)^i} + b_t \quad (3.2)$$

Where b_t is the value of bubble component at time t such that:

$$p_t = p_t^* + b_t \text{ where } E_t(b_{t+1}) = (1+r)b_t \quad (3.3)$$

This implies that rational speculative bubbles can exist in financial markets as long as the rate of growth of the value of the bubble is equal to its discount factor. The price of the asset including the bubble is then still an equilibrium value and investors can rationally invest in it as long as they believe that the bubble will grow at the discount rate r .

3.1. Tests for asset price bubbles

There are a number of approaches used to test empirically in the literature for the presence of rational speculative bubbles in asset prices.

Relationship models look at statistical relationships that exist for the assets in question and their fundamental driver. Shiller (1980) uses the variance bound tests of the equity prices to show that their variance is too large to be justified by fundamentals. Tests for long run relationships between prices and fundamentals use Unit Roots and Cointegration tests. The Markov Switching ADF test used in this paper represents an advanced version of these tests.

Phillips, Wu, and Yu (2011) provide another modern method within this general approach, the sup-ADF. They use forward recursive right-tailed ADF tests and state that this method can also be used to anticipate bubbles, making it very

useful for policy makers. [Homm and Breitung \(2012\)](#) test the power of [Philips et al.'s \(2011\)](#) sup-ADF against a number of alternatives including a version of the sup-ADF test including a chow test modification, the sup-ADFC. They find that for randomly starting bubbles their new sup-ADFC test outperforms the sup-ADF in terms of bubble detection. However the sup-ADF is by far the most powerful in detecting periodically collapsing bubbles.

Counting models, also known as hazard models, are different in that they do not compare the time series behaviour of the determining factors of the value of the asset with its price. These models include [McQueen and Thorley \(1994\)](#) non-parametric duration dependence test which they applied to equity markets.

Explicit models such as [Wu \(1997\)](#) treat bubbles strictly as deviations from the present value model shown in equation (3.2), allowing the bubble to be estimated as a time series variable. The weakness of this approach and relationship models is common to much finance research. We are really testing a joint hypothesis; is there a bubble and is our model of the assets price correct which is similar to the problems found when testing EMH ([Lo, 2007](#)). Any misspecification of the present value model is included in the bubble component so that it cannot be shown decisively if a bubble is present or the model used by the researcher needs correction.

3.2. Bubbles in gold prices

3.2.1. Early models

[Diba and Grossman \(1984\)](#) form an equation for the price of gold based on an investor's portfolio demand for gold composed of three parts.

$$s_t + p_t = \beta(E_t(p_{t+1}) - p_t) - \gamma E_t r_{t+1} + o_t \quad (3.4)$$

Where: p_t is the log of the gold price, s_t is the log of the stock of gold at t , β is a positive constant, showing the relationship between the portfolio demand for gold and the real return on gold, γ is a positive constant, showing the relationship between the portfolio demand for gold and the real return on other assets, $E_t(\cdot)$ denotes the rational expectations operator, r_{t+1} represents the rate of return on other assets, o_t is other factors that affect gold's fundamental value that are not observable.

The fundamental component (FC) of the value of gold is given by 3.4 above. It states that the total value of the stock of gold is based on what is expected to happen to its price in the future as well as being negatively related to what you can earn on other assets. [Diba and Grossman \(1984\)](#) use real interest rates in their model as the return on other assets. Here we will also include the lease rate on gold as another reason to hold gold other than expected price changes. Gold lease rate data begins in 1989 and therefore fell into the u_t category of unobservable variables in their 1984 model.

The other parts of what determines the price of gold are the Stochastic Bubble Component (SBC), a random variable with a zero mean whose value falls to zero as time progresses and

the deterministic bubble component (DBC) is what we are looking for in testing for the presence of bubbles here and if it is found to be present then we have a rational bubble. These are shown in equation (3.5) below.

The DBC is a constant, times an eigenvalue raised to a power greater than 1 $[(1 + \beta^{-1})^t]$. This implies that as t increases the DBC increases. The SBC is a constant, times an eigenvalue raised to a power less than one, so that it decreases with t $[(1 + \beta^{-1})^{-t}]$. Their equation for the time path of the price of gold is shown below in equation (3.5).

$$p_t = (1 + \beta)^{-1} \sum_{i=0}^{\infty} (1 + \beta^{-1})^{-i} E_t(u_{t-i} - \gamma r_{t+1-i} - s_{t+i}) \\ + c(1 + \beta^{-1})^t + \sum_{i=1}^t (1 + \beta^{-1})^{t-i} z_i FC DBC SBC \quad (3.5)$$

Where c is a constant determined by an initial condition and z_i is a random variable representing new information with a zero mean and is uncorrelated with all variables. In the analysis z_t is treated as an unobserved variable.

[Diba and Grossman \(1984:8\)](#) state that "the intuitive distinction between FC and the bubble components is that, if the market collectively misunderstands FC, individuals can gain by contradicting the market, whereas if the market does not expect a price bubble, individuals who act on the basis of price forecasts incorporating a bubble will lose". The bubble components are rational when the market collectively incorporates them in price forecasts.

From this equation the stationarity properties of the process that generates p_t can be investigated for evidence for or against bubbles. As we cannot observe the DBC we must make inferences about the process that generates the DBC. If the c equation (3.5) is non-zero it will be mean that the DBC is non-stationary as it grows at $(1 + \beta^{-1})^t$, regardless of how many times it is differenced ([Gurkaynak, 2008](#)). If we find that the process generating the FC components is stationary, p_t would also be stationary if no bubble is present.

The number of times it is necessary to difference the determinants of gold's value to make them stationary would then be the number of times it is necessary to make p_t stationary, if p_t is the fundamental value and is determined by its lease rate and leasing cashflows. As [Evans \(1991\)](#) argues if the price series of an asset is not more explosive than its fundamental determinant then it can be said that no bubble is present, as the fundamental component is what gives us the price series.

[Diba and Grossman \(1988\)](#) test for a bubble in the price of shares using the idea that if two series are found to be I(1) from both sets of ideas above, and their linear combination cointegrates, then there is an equilibrium relationship between them, implying that no bubbles exist. They argue that it is unlikely that an unobserved fundamental will be I(2), meaning that failing to reject a cointegrating relationship for variables is proof of a fundamentally determined price. Rejection however may not prove that a bubble exists due to differing power and size properties of cointegration tests.

3.2.2. Periodically bursting bubbles

Evans (1991) points out that these tests are only applicable for bubbles that continue to grow in p_t from $t = 0$ as the c component is not time varying and must be present from the start in order to enter the series. They do not have any ability to detect periodically bursting bubbles. Evans also assumes that a bubble cannot be negative, but unlike earlier work bubbles in this model can now collapse to a low but positive value. The bubble can then be in one of two different states at any time.

$$B_{t+1} = (1+r)B_t U_{t+1} \text{ if } B_t \leq \alpha \quad 3.7a$$

$$B_{t+1} = [(\delta + \pi^{-1}(1+r)\theta_{t+1}) * [B_t - (1+r)^{-1}] * U_{t+1} \text{ if } B_t > \alpha \quad 3.7b$$

Where δ and α are positive parameters such that $0 < \delta < (1+r)\alpha$ and U_{t+1} is an exogenous identically and independently distributed (iid) random variable with $\Sigma(U_{t+1}) = 1$. θ is an exogenous and Independently identically distributed (IID) Bernoulli process independent of U_t which takes on a value of 1 with a probability of π and a value of 0 with a probability of $1-\pi$.

Thinking of bubbles in this way increases our ability to identify them, as under Diba and Grossman (1988) model the bubble would need to be present over the period being examined. Evans (1991) points out that it is more likely that bubbles appear and disappear, making it more likely that the process will appear stationary but in reality still contain speculative bubbles. Testing for this class of bubbles represents a more realistic test of what we would expect to see in reality.

Hall, Psaradakis, and Sola (1999) use simulation and an empirical example to put Evans (1991) idea into practise using a Markov Switching Augmented Dickey–Fuller (MSADF) following Hamilton's (1989) method. This method has been applied to bubble detection as in Liu, Margaritis, and Wang (2012) and in other contexts such as studying real exchange rate nonlinearities in Kruse, Frömmel, Menkhoff, and Sibbertsen (2012).

3.2.3. Evidence for bubbles in the price of gold

Diba and Grossman (1984) find that the price of gold is entirely based on market fundamentals using conventional unit root and cointegration tests with real interest rates on commercial paper as the measure of gold opportunity cost. Evans (1991), as previously discussed, criticised these results on the basis that they do not detect periodically bursting bubbles.

A number of researchers have used gold's CY to find a true fundamental value for gold, in the same way as is normal for storable consumption commodities, such as oil or copper. Pindyck (1993) assumes that the fundamental value of a commodity is the present value of expected future payoffs. A gold price bubble is found between 1975 and 1990, but when it occurs cannot be specified. Went et al. (2009) find evidence of a bubble using a duration dependence test on the monthly interest-adjusted basis, a measure of the potential excess returns earned on commodities through their CY. Bialkowski

et al. (2011) find the deviations of gold price from its fundamental value based on a CY approach and they see no evidence of a bubble in the period between 1978 and 2010.

Bertus and Stanhouse (2001) use dynamic factor analysis to look for bubbles in the quarterly futures price of gold. They build an explicit model of the supply and demand for gold, based on the macroeconomic drivers discussed in Section 2.2, to derive a fundamental price and use this to estimate a time series for the bubble component in the price. The bubble component is however found to be insignificant so they conclude that no bubble is present. Bialkowski, Bohl, Stephan, and Wisniewski (2012) build an approximation of gold's fundamental value based on a similar set of macroeconomic drivers and apply a Markov-Switching ADF test. They find no evidence of a bubble when the European sovereign debt crisis is accounted for. This however points to a weakness of this approach, over-specification. If enough explanatory variables are included not bubble finding is not much of a surprise. Baur and Glover (2012) also argue that some of the variables used in Bialkowski et al. (2012) do not represent fundamental drivers but factors that attract speculative investors, which would cause the bubble.

Baur and Glover (2012) apply Phillips et al. (2011) sup-ADF tests (forward recursive ADF tests) for explosive price behaviour but do make any assumptions about the fundamental determinant of the value of gold. They conclude that the gold price has been in a bubble between 2002 and 2012, except in 2008–9 during the sub-prime mortgage crisis due to its explosive price behaviour.

4. Methodology

4.1. Unit roots and cointegration

Diba and Grossman (1984) test for unit roots in gold prices by looking at the Auto Correlation Function of the gold price and real interest rates, as well as their 1st and 2nd differences. Diba and Grossman (1988) use Augmented Dickey–Fuller (ADF) tests to provide a more rigorous way of looking for unit roots in the variables, as employed by and shown in equation (4.1) below.

$$\Delta y_t = \pi y_{t-1} + \sum_{j=1}^p \beta_j y_{t-j} + \varepsilon_j \quad 4.1$$

Where y is the asset price and $\Delta y_t = y_t - y_{t-1}$.

Diba and Grossman (1988) also test for cointegrating relationships between gold and its fundamental determinants. Following the earlier ADF tests they estimate:

$$\Delta v_t = \pi v_{t-1} + \sum_{j=1}^k \beta_j \Delta v_{t-j} + \varepsilon_j \quad 4.2$$

where v_t are the residuals from the regression of asset prices on the relevant fundamental determinant. We run these as preliminary tests.

Table 1
Descriptive statistics.

	Fixing		Lease rate				
	AM	PM	1 month	2 month	3 month	6 month	12 month
Observations	6283	6283	6283	4075	6283	6283	6283
Mean	576.62	576.21	0.6731	0.4343	0.7706	0.8806	1.0756
Standard Deviation	410.04	409.62	0.8502	0.7342	0.8292	0.7888	0.7708
Skewedness	1.6719	1.6730	1.9708	2.9841	1.6595	1.3116	0.8330
Kurtosis	1.5878	1.5923	7.5074	15.978	5.5460	3.0566	1.2623

Source: LBMA website.

4.2. Markov Switching ADF

Hall et al. (1999) generalise these tests of asset prices using a Markov Switching ADF framework, as discussed in Hamilton (1989, 1990). This provides a development of the ADF tests described above to take into account the critique of Evans (1991), that traditional unit root and cointegration tests do not account for periodically collapsing bubbles.

We adjust equation (4.1) as in Hall et al. (1999) so that it is now time varying, changing with an unobserved indicator s_t , the stochastic regime variable, which takes on a value of 0 or 1 so that:

$$\Delta y = \pi_{s_t} + \Phi_{s_t} y_{t-1} + \sum_{i=1}^k \beta_{s_t,i} \Delta y_{t-i} + \sigma_{s_t}^2 \varepsilon_t \tag{4.3}$$

Where $\pi_{s_t} = \pi_0 + s_t(\pi_1 - \pi_0)$, $\Phi_{s_t} = \Phi_0 + s_t(\Phi_1 - \Phi_0)$, $\beta_{s_t,i} = \beta_{0,i} + s_t(\beta_{1,i} - \beta_{0,i})$ and $\sigma_{s_t}^2 = \sigma_0^2 + s_t(\sigma_1^2 - \sigma_0^2)$. ε_{s_t} is normally distributed and k is the lag order of the model. We assume that the probability that the process is in a particular regime at time t depends only upon the probability of which regime the process was in at time $t - 1$, and not on earlier periods. We therefore model this random sequence to a homogenous Markov chain with transition probabilities as defined as below, allowing the data to determine whether or not we are in a particular state:

$$\begin{aligned} \Pr(s_t = 1 | s_{t-1} = 1) &= p \\ \Pr(s_t = 0 | s_{t-1} = 1) &= 1 - p \\ \Pr(s_t = 1 | s_{t-1} = 0) &= q \\ \Pr(s_t = 0 | s_{t-1} = 0) &= 1 - q \end{aligned} \tag{4.4}$$

From this we will also find the probability that the series is in regime 1 at any time (p) or in regime 2 ($1 - p$), the transition probabilities.

We estimate the parameters using maximum likelihood procedures as in Hall et al. (1999) to test the null hypothesis of a bubble in either regime (i.e. that $\Phi_{s_t} > 0$, a right tailed test for explosiveness) against the alternative of a regime which is either stationary or has a unit root. If $\Phi_{s_t} > 0$ in any state we can say that a bubble is present in the data for that state. The process gives inferences about the probability of being in a particular state [$\Pr(s_t = 1 | I_t, \vartheta)$], where $I_t = (y_t, \dots, y_1)$ and $\vartheta = (\pi_1, \pi_2, \Phi_1, \Phi_2, \beta_{0,1} \dots \beta_{0,k}, \beta_{1,1} \dots \beta_{1,k})$, the filter probabilities that we are in state 1 at any time. These allow us to make inferences about which of the unobserved regimes we are in at any time as in Hamilton (1989) and Hall et al. (1999). We use the Broyden–Fletcher–Goldfarb–Shanno (BFGS) algorithm following Shi (2012) who finds this outperforms the alternative expectation-maximization (EM) algorithm in estimating this type of model.

We can also allow the variance in each regime to switch between the two states, as in Brooks and Persaud (2001) or remain constant as in Shi (2012). Shi (2012) argues that imposing a constant variance can improve the power of the

Table 2
Maximum likelihood estimates: Residuals of PM fixing and lease rates: MSADF-RV.

	1 Month	2 Month	3 Month	6 Month	12 Month
Φ_1	-0.0104 [-1.742]	-0.0001 [-0.010]	-0.0046 [-0.981]	-0.005 [-0.891]	-0.0053 [-1.003]
Variance 1	0.6215***	0.6436***	0.4814***	0.4503***	0.3460***
Φ_2	-0.0045*** [-4.609]	-0.0061*** [-5.579]	-0.003*** [-5.695]	-0.002*** [-3.956]	-0.0014 [-2.345]
Variance 2	0.1073***	0.0952***	0.1090***	0.1028***	0.0996***
P_{12}	0.2883***	0.2636***	0.2550***	0.2581***	0.3424***
P_{21}	0.0694***	0.0464***	0.0579***	0.0472***	0.0456***

Note: *, ** and *** represent significance at the 10%, 5% and 1% level, t-stats are given below in parentheses.

Table 3
Maximum likelihood estimates: residuals of PM fixing and lease rates: MSADF-CV.

PM fixing and lease rates: daily data					
	1 Month	2 Month	3 Month	6 Month	12 Month
Φ_1	-0.007 [-0.258]	0.225*** [8.687]	0.256*** [8.807]	0.0458 [1.383]	0.1071* [2.196]
Φ_2	-0.004*** [-5.646]	-0.010*** [-0.4893]	-0.005*** [-6.607]	-0.0046*** [-5.293]	-0.0032*** [-3.999]
P_{12}	0.925***	0.775***	0.814***	0.639***	0.718***
P_{21}	0.018***	0.015***	0.010***	0.007***	0.010***

Note: *, ** and *** represent significance at the 10%, 5% and 1% level, t-stats are given below in parentheses.

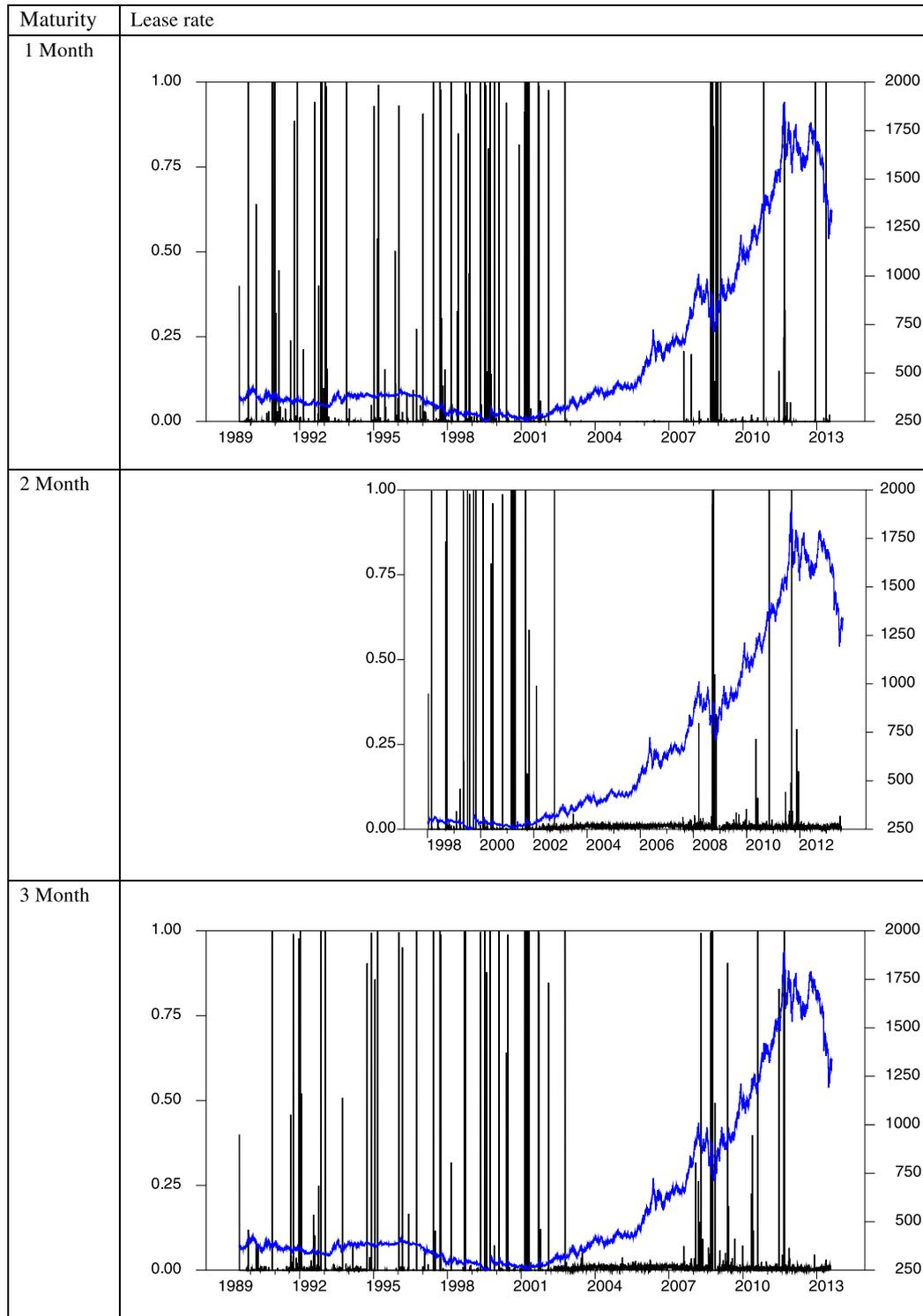


Fig. 3. The probability of being in regime 1 at t , MSADF-CV.

test, though it may lead to the rejection of the null too often when the differences between the residual variance of the different regimes is large. It also makes the results more robust to model misspecification. We carry out both a constant variance (MSADF-CV) and regime switching variance ADF (MSADF-RV) test to assess the sensitivity of our results to

these factors but more emphasis is put on the MSADF-CV due to its increased power.

Nelson, Zivot, and Piger (2001) chose their lag length using the backward lag-length selection procedure as found in Campbell and Perron (1991) with the lag length, k , set equal to a maximum of the lower integer bound of $T^{1/3}$ as proposed in

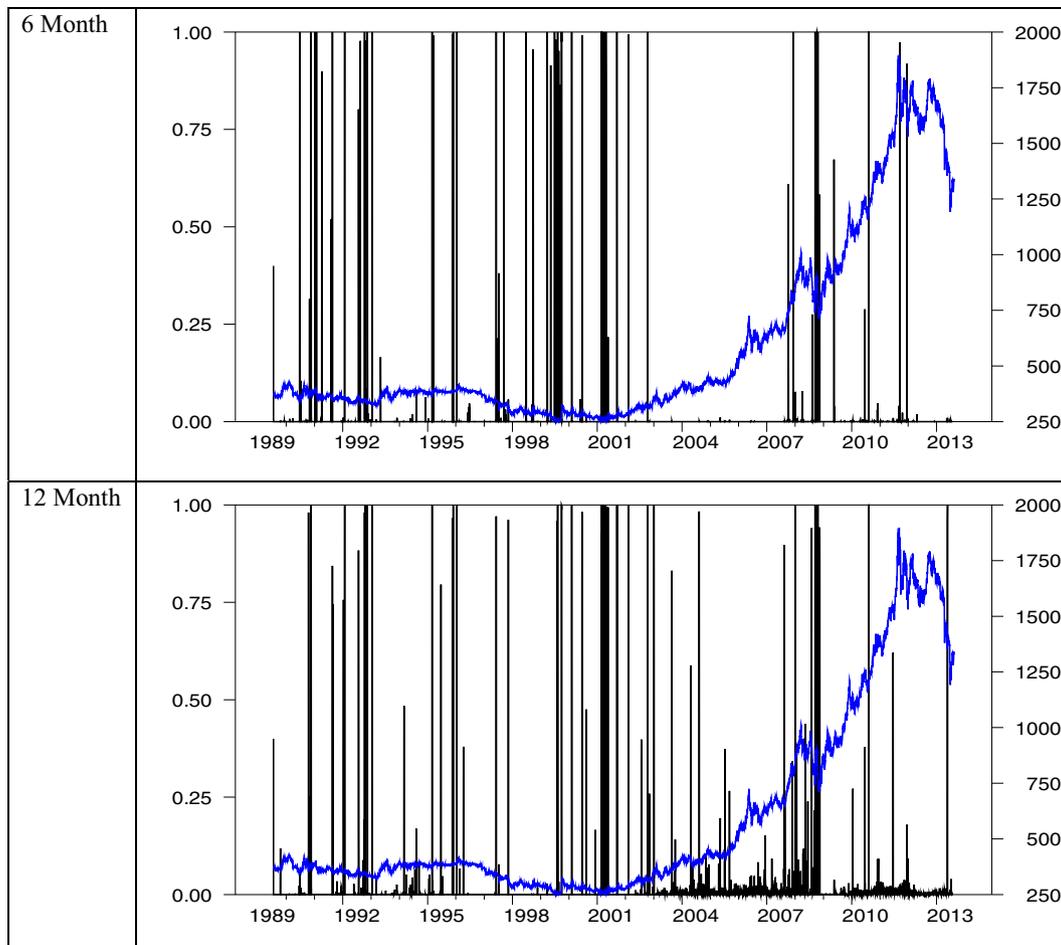


Fig. 3. (continued).

Said and Dickey (1984). Camacho (2011) uses the AIC and SCB in formations criteria. Here we apply the three methods and when they disagree, we test all suggested models only reporting the most parsimonious model if the results are qualitatively the same.

Camacho (2011) points out that the distribution of the t -statistic used to test the null is nonstandard. We follow Hall et al. (1999) by bootstrapping the model under the null hypothesis in order to calculate the simulated critical values, using the estimates of the model parameters from this realisation of y_t . The number of bootstrap replications is 50,000. The steps of the estimating procedure are given in Camacho (2011).

5. Data

Table 1 below shows the descriptive statistics for the daily data used in this study. The AM and PM fixings are daily over the counter London spot prices. Lucey, Larkin, and O'Connor (2013) analyse whether the price of gold is set through the Fixings in London or the futures market in New York, finding that both play a dominant role at different times. All data is available from 17th of July 1989 up to the 31st of July 2013 for all variables except the 2 month lease rate which begins on

the 2nd of January 1998. The lease rates are all annualised figures. Data is available from the London Bullion Market Association (LBMA) website.

6. Results

6.1. Preliminary tests: traditional ADF and cointegration tests

The AM, PM Fixings and lease rates are $I(1)$ in levels and $I(0)$ in 1st differences based on ADF tests. No cointegrating relationships were found to exist between any of the $I(1)$ pairs using traditional cointegration tests. The lack of a cointegrating relationship between the variables implies that a long run equilibrium relationship may not exist between gold prices and gold lease rates, so that *rational speculative bubbles* occur in the price of gold. However as these methods' ability to detect bubbles is very poor in the presence of periodically bursting bubbles, as discussed in Section 4.2, the results are not reported here¹ and are discussed purely as a preliminary to the results of test for *periodically collapsing bubbles* in the next section.

¹ These are available from the authors on request.

6.2. Markov-Switching Augmented Dickey–Fuller tests for periodically bursting bubbles

In Tables 2 and 3 below the results of both the MSADF-RV and MSADF-CV models derived from equation (4.3) are shown, for the residuals of the PM Fixing and each lease rate maturity. The results for the AM fixings are not shown as they are qualitatively identical. Φ_1 and Φ_2 are the means of regime 1 and 2 respectively, Variance 1 and Variance 2 are the regimes respective variances (where applicable) and P_{12} and P_{21} are the probability of switching from regime 1 to regime 2 and vice versa. All estimations include a constant term and the appropriate number of lags as determined from the procedures discussed in Section 4.2 but the coefficients are not reported here.

We can see in Table 2 that when we allow the variance to differ across regimes (MSADF-RV) two regimes exist for all lease rate maturities. However regardless of the lease rate maturity no bubble is found to be present. Based on a right tailed test for explosiveness with a null of $\Phi_{st} > 0$ all equations fail to find a bubble. Regime 1 is stationary, i.e. Φ_1 is not significantly different from zero, while regime 2 has a unit root, but it is not explosive.

We find that the two regimes are characterised not by one having a bubble but by sizeably different variances. Regime 1 has the lower variance by a multiple of between approximately 3 and 6 times depending on which maturity of lease rate used as the fundamental determinant in the equation. The residuals of the relationships between the gold price and measures of the benefit of holding gold are then characterised by periods of stability (regime 1) and increased volatility (regime 2), that is times when lease rates are not as powerful an explanatory variable for gold prices.

The results of the estimations when we restrict the variance to be the same in both regimes (MSADF-CV) are shown in Table 3. This approach finds in favour of a bubble in the gold price when the 2, 3 and 12 month lease rates are used as the fundamental determinant of gold prices. Φ_1 is significantly greater than 0 at the 1% level for 2 and 3 month maturity and at 10% for 12 months, indicating an explosive unit root and so a periodically collapsing bubble in the gold price. For 1 and 6 month maturities no bubble is found as regime 1 is stationary and regime 2 is not explosive. The probability of switching from the bubble regime to regime 2 is very high and once in regime 2, it switches much more infrequently. As these results are more robust, based on Shi's (2012) work, we focus on these below.

Fig. 3 plots the inferred filter probabilities of being in regime 1 at any given time based on the information available at that time, as in Hamilton (1989), for all maturities under the MSADF-CV estimations. When the probability is less than 0.5 we are in regime 1 and when greater than 0.5 we can say that we are probably in regime 2. From this we can see a long period of stability in the mid 2000's where the gold price was fundamentally justified based on the 2 and 3 month lease rates. The relationship between gold prices and lease rates frequently switches into a bubble over the rest of the sample but quickly moves out of it each time.

Table 4 shows the cross-correlation matrices for the filter probabilities for all lease rates based on the MSADF-CV. We see the highest correlation between the 2 and 3 month estimations where we find a bubble with the highest degree of confidence.

7. Conclusions

In attempting to answer whether bubbles occur in the price of gold we use gold lease rates as a measure of gold's fundamental value for the first time in the literature, testing for rational speculative bubbles and periodically bursting bubbles using two types of regime switching models.

The traditional unit root and cointegration for rational bubbles give mixed evidence, but indicate the possibility of a rational speculative bubble in the price of gold. However these tests have a poor ability to deal with periodically bursting bubbles as explained by Evans (1991). We therefore move onto using Markov Switching ADF tests for cointegration to allow the assumption of a single long run relationship to be relaxed. Two different regimes are allowed to exist to assess whether bubbles form and then burst over time.

We run these tests to allow for the variance to be a switch between the two regimes and holding it constant, as recommended by Shi (2012). Allowing the variance to switch results in no bubble being found. Instead the two regimes are characterised by a high and low variance; this indicates that in the high variance regime the lease rates' power as explanatory variables decreases significantly. When we restrict the variance between regimes we do find some evidence of a bubble using the 2, 3 and 12 month lease rates. A stable period in the mid-2000's shows the gold price as fundamentally determined by its lease rates but at other times bubble phases occur frequently if only for short periods. As the constant variance model is more powerful and robust we must conclude that there is some evidence of bubbles in gold prices.

Further research in applying lease rates to gold prices as a fundamental determinant to assess whether bubbles occur are necessary to get a clearer picture, asking the question using different approaches such as Philips et al.'s (2011) sup-ADF test or McQueen and Thorley (1994) non-parametric duration dependence test for bubbles. This paper's use of a market based measure of gold's economic benefit however does provide new and theoretically strong evidence that gold has been through some bubble phases at certain times over the last 20 years.

Table 4
Cross-correlation matrix: probabilities of being in regime 1, for all maturities MSADF-CV.

	1 month	2 month	3 month	6 month	12 month
1 Month	1	–	–	–	–
2 Month	0.556	1	–	–	–
3 Month	0.491	0.727	1	–	–
6 Month	0.500	0.556	0.565	1	–
12 Month	0.399	0.480	0.534	0.644	1

References

- Barone-Adesi, G., Geman, H., & Theal, J. (2010). *On the lease rate, convenience yield and speculative effects in the gold futures market*. Swiss Finance Institute Research Paper No. 09–07.
- Baur, D., & Lucey, B. (2010). Is gold a hedge or a safe haven? An analysis of stocks, bonds and gold. *Financial Review*, 45(2), 217–229.
- Baur, D., & McDermott, T. (2010). Is gold a safe haven? International evidence. *Journal of Banking and Finance*, 34(8), 1886–1898.
- Baur, D., & Glover, K. (2012). *A gold bubble?*. UTS Business School Working Paper no. 175.
- Bertus, M., & Stanhouse, B. (2001). Rational speculative bubbles in the gold futures market: an application of dynamic factor analysis. *The Journal of Futures Markets*, 21(1), 79–108.
- Bialkowski, J., Bohl, M., Stephan, P., & Wisniewski, T. (2011). *Is there a speculative bubble in the price of gold?*. SSRN Working Paper.
- Bialkowski, J., Bohl, M., Stephan, P., & Wisniewski, T. (2012). *The gold price in times of crisis*. SSRN Working Paper.
- Brooks, C., & Persaud, G. (2001). The trading profitability of forecasts of the gilt–equity yield ratio. *International Journal of Forecasting*, 17, 11–29.
- Camacho, M. (2011). Markov-switching models and the unit root hypothesis in real US GDP. *Economics Letters*, 112, 161–164.
- Campbell, J., & Perron, P. (1991). Pitfalls and opportunities: what macroeconomists should know about unit roots. *NBER Macroeconomics Annual*, 141–201.
- Christie-David, R., Chaudhry, M., & Koch, T. (2000). Do macroeconomic news releases affect gold and silver prices. *Journal of Economics and Business*, 52, 405–421.
- Coudert, V., & Feingold, R. (2011). Gold and financial assets: are there any safe havens in bear markets? *Economics Bulletin*, 31(2), 1613–1622.
- Denning, S. (2012). *Why Warren Buffett won't invest in gold*. Forbes.com. Available at <http://www.forbes.com/sites/stevedenning/2012/05/09/why-warren-buffett-wont-invest-in-gold> Accessed 15.11.12.
- Diba, B., & Grossman, H. (1984). *Rational bubbles in the price of Gold*. NBER paper no. 1300.
- Diba, B., & Grossman, H. (1988). Explosive rational bubbles in stock prices? *The American Economic Review*, 78(3), 520–530.
- Evans, G. (1991). Pitfalls in testing for explosive bubbles in asset prices. *American Economic Review*, 81(4), 922–930.
- Ghosh, D., Levin, E., Macmillan, P., & Wright, R. (2004). Gold as an inflation hedge. *Studies in Economics and Finance*, 22, 1–25.
- GFMS Gold Survey, 2012. London: Thomson Reuters.
- Gurkaynak, R. (2008). Econometric tests of asset price bubbles: taking stock. *Journal of Economic Surveys*, 22(1), 166–186.
- Hall, S., Psaradakis, Z., & Sola, M. (1999). Detecting periodically collapsing bubbles: a Markov switching unit root test. *Journal of Applied Econometrics*, 14, 143–154.
- Hamilton, J. (1989). A new approach to the economic analysis of nonstationary time series and the business cycle. *Econometrica*, 57, 357–384.
- Hamilton, J. (1990). Analysis of time series subject to changes in regime. *Journal of Econometrics*, 45, 39–70.
- Homm, U., & Breitung, J. (2012). Testing for speculative bubbles in stock markets: a comparison of alternative methods. *Journal of Financial Econometrics*, 10(1), 198–231.
- Hull, J. (2006). *Options, futures and other derivatives* (6th ed.). Prentice Hall.
- Kaufman, T., & Winters, R. (1989). The price of gold: a simple model. *Resource Policy*, 15(4), 309–313.
- Koutsoyiannis, A. (1983). A short-run pricing model for a speculative asset, tested with data from the gold bullion market. *Applied Economics Letters*, 15, 563–581.
- Kruse, R., Frömmel, M., Menkhoff, L., & Sibbertsen, P. (2012). What do we know about real exchange rate nonlinearities? *Empirical Economics*, 43(2), 457–474.
- Kutan, A., & Aksoy, T. (2004). Public information arrival and gold market returns in emerging markets: evidence from the Istanbul Gold Exchange. *Scientific Journal of Administrative Development*, 2, 13–26.
- Lawrence, C. (2003). *Why is gold different from other assets? An empirical investigation*. London: World Gold Council.
- LBMA (London Bullion Market Association). (2008). *A guide to the London precious metals markets*. The London Bullion Market Association and the London Platinum and Palladium Market.
- Liu, X., Margaritis, D., & Wang, P. (2012). Stock market volatility and equity returns: evidence from a two-state Markov-switching model with regressors. *Journal of Empirical Finance*, 19(4), 483–496.
- Levin, E., Abhyankar, A., & Ghosh, D. (1994). Does the gold market reveal real interest rates? *Manchester School (supplement)*, 62, 93–103.
- Levin, E., & Wright, D. (2006). *Short-run and long-run determinants of the price of gold*. World Gold Council. Research Study No. 32.
- Lo, A. (2007). Efficient Markets Hypothesis. In L. Blume, & S. Durlauf (Eds.), *The new Palgrave: A dictionary of economics* (2nd ed.). (Palgrave Macmillan Ltd).
- Lucey, B., Larkin, C., & O'Connor, F. (2013). London or New York: where and when does the gold price originate? *Applied Economics Letters*, 20(8), 813–817.
- Lucey, B., & O'Connor, F. (2013). *A behavioural examination of the forward rate unbiasedness hypothesis in precious metals markets*. SSRN Working Paper.
- McQueen, G., & Thorley, S. (1994). Bubbles, stock returns, and duration dependence. *Journal of Financial and Quantitative Analysis*, 29(3), 379–401.
- Nelson, C., Zivot, E., & Piger, J. (2001). Markov regime switching and unit root tests. *Journal of Business & Economic Statistics*, 19(4), 404–415.
- O'Connor, F., & Lucey, B. (2012). Gold's negative relationship with the US dollar. *The Alchemist*, (64).
- Pindyck, R. (1993). The present value model of rational commodity pricing. *Economic Journal*, 103(418), 511–530.
- Phillips, P., Wu, Y., & Yu, J. (2011). Explosive behavior in the 1990s Nasdaq: when did exuberance escalate asset values? *International Economic Review*, 52(1), 201–226.
- Said, S., & Dickey, D. (1984). Testing for unit roots in autoregressive-moving average models of unknown order. *Biometrika*, 71, 599–607.
- Sari, R., Hammoudeh, S., & Soytas, U. (2010). Dynamics of oil price, precious metal prices, and exchange rate. *Energy Economics*, 32(2), 351–362.
- Shi, S. (2012). Specification sensitivities in the Markov-switching unit root test for bubbles. *Empirical Economics*, 1–17.
- Shiller, R. (1980). Do stock prices move too much to be justified by subsequent changes in dividends? *The American Economic Review*, 71(3), 421–436.
- Tully, E., & Lucey, B. (2007). A power GARCH examination of the gold market. *Research in International Business and Finance*, 21, 316–325.
- Vissing-Jørgensen, A. (2003). Perspectives on behavioural finance: does irrationality disappear with wealth? Evidence from expectations and actions. *NBER Macroeconomics Annual*, 18.
- Went, P., Jirasakuldech, B., & Emekter, R. (2009). *Bubbles in commodities markets*. SSRN Working Paper.
- Wu, Y. (1997). Rational bubbles in the stock market: accounting for the U.S. stock-price volatility. *Economic Inquiry*, 35, 309–319.